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Final reheating temperature on a single brane

Rouzbeh Allahverdi [1], Anupam Mazumdar [2] and A. Pérez-Lorenzana [2,3]
[1] *Physik Department, TU Muenchen, James Frank Strasse, D-85748, Garching, Germany.*
[2] *The Abdus Salam International Centre for Theoretical Physics, I-34100, Trieste, Italy.*
[3] *Departamento de Física, Centro de Investigación y de Estudios Avanzados del I.P.N. Apdo. Post. 14-740, 07000, México, D.F., México.*

Abstract

We make a generic remark on thermal history of a single brane cosmology in models with an infinitely large single extra dimension. We point out that the reheat temperature of the Universe is bounded by an excess production of gravitons from the thermal bath. The actual bound is given by the brane tension. If the initial temperature of the Universe is larger than this bound, then an efficient graviton production shall prevail. However, the brane cools down gradually as the Kaluza-Klein gravitons take the energy in excess away from the brane. The cooling continues until the radiation dominated phase is restored, which occurs before the Big Bang Nucleosynthesis. We argue whatsoever be the early evolution of the Universe, the final radiation dominated phase always starts after the Universe transits from non-conventional era to the standard cosmological era.

There has been much interest in the possible existence of a non compact (infinite) extra dimension [1]. This has a striking feature that a four dimensional gravity can be thought of as being a zero mode of a $4+1$ dimensional anti de-Sitter bulk field which is localized on a hypothetical $3+1$ dimensional Poincaré invariant brane where we are assumed to live. This gravitational zero mode has a unique profile which decays as we go away from the brane due to the presence of a non-trivial warp factor which peaks around the brane. A simple extension of this proposal can also solve the hierarchy between the Planck scale and the electroweak scale, if another brane with a negative tension is located at a distance where the vacuum expectation value of the Higgs naturally picks up an electroweak scale due to the presence of the warp factor [2]. It has been noted earlier in Ref. [3], that the thermal history of such a Universe departs significantly from the standard lore. This is mainly due to the fact that the $(0,0)$ component of the Einstein's equation contains some new terms which are present due to the fact that the brane is infinitely thin, and, the matter fields in the $4+1$ dimensional set-up are actually supposed to be localized on such a brane [4,5]. An interesting feature of such a brane world system is a continuous mass spectrum of the

Kaluza-Klein (KK) modes of gravity expanding from zero mass up to the Planck scale (for a review, see Ref. [6]). It means, from the five dimensional point of view the gravitons can take away any amount of energy in the form of the fifth momentum. This is in contrast with the usual KK theories and all kind of compact extra dimensional models, where the KK spectrum is always discrete (see for instance, Refs. [7–11]).

It is usually believed that these KK modes can be excited from a thermal bath once inflation comes to an end. As the space is compact they just circulate around maintaining their influence on the brane. Eventhough, these KK modes are weakly coupled to other matter fields, they could pose a threat to the synthesis of light elements in a similar way as in the case of a massive unstable relic particle [12]. Although, this is a particular feature in models with compact extra dimensions, it is not considered to be true in the present case, because the KK gravitons can truly leave the brane into the extra infinite dimension. From the point of view of a four dimensional observer they notice a loss of energy from the brane. This might be helpful because the dangerous KK modes may no longer threat nucleosynthesis whatsoever.

Usually, the reheat temperature of the Universe is recognized as the largest temperature of the Universe during the radiation epoch that extends up to the nucleosynthesis era. In almost all the cases the reheat temperature of the Universe is constrained in order not to over produce weakly coupled particles such as KK gravitons, gravitinos and moduli. Especially in the supersymmetric case one has to worry about the gravitinos because their mass is ~ 1 TeV in gravity mediated supersymmetry breaking models, and their coupling to other fields is Planck mass suppressed. This is the reason why they decay during the nucleosynthesis era. When they decay they inject entropy and reheat the plasma [13]. Very similarly once the KK modes are produced they go out of equilibrium from the rest of the thermal bath and their number density redshifts as the Universe expands. The mass of these modes remain same when they decay and their decay products may inject entropy to the Universe before nucleosynthesis, during nucleosynthesis, and after nucleosynthesis. If they decay much later, then, they can be constrained from the diffusion of photons in a micro wave background radiation [11,14].

In this paper we discuss thermal history of a single brane cosmology. We argue that the reheat temperature of a brane is only bounded by a threshold temperature, known as the *normalcy temperature* of the KK gravitons. This temperature determines the departure from the radiation dominated Universe to a KK dominated Universe. Beyond this temperature apart the usual cooling due to the expansion, there is an extra source of cooling. The brane can also radiate the excess of energy to the extra space. This cooling process becomes less significant when the temperature of the brane becomes as low as the normalcy temperature, since below this temperature the KK graviton production is not anymore efficient. In what follows we briefly introduce the cosmology of a single brane. We then study the abundance of gravitons and thermal history of the Universe in a more general context. For consistency we treat gravitons always in terms of KK modes, although eventually the five dimensional point of view helps to understand their dynamics in the bulk.

The cosmology of a single brane is modified significantly due to the fact that the energy momentum tensor is localized on a brane with $T^{\mu}_{\nu\ brane} = \delta(y)\,(-\rho, p, p, p, 0)$. This defines an appropriate boundary condition for the cosmological parameters in the extra spatial direction and changes the Friedmann equation while describing the time dependent scale factor on the brane [4,5,15,16]. In the simplest scenario, where the extra dimension is supposed to be *stable* and free from *space-like singularities*, no time dependent contribution comes from the bulk. In particularly, an extra term appears in the Friedmann equation which goes as $\sim 1/a^4$, where a is the time dependent scale factor on the brane [4]. Such a term is usually interpreted as a dark radiation contribution to the brane, and actually encodes the information of the time dependence of the fifth dimension [5,17]. Also, it has been conjectured on the basis of AdS/CFT correspondence that such a time dependent source term might appear due to the presence of a black hole in the bulk (see for instance, Refs [18,19]). However, as it has been shown in Ref. [17], such a term is absent if we assume that the bulk is stable. In such a case the Friedmann equation simplifies and yields

$$H^2 = \frac{8\pi}{3M_{\rm p}^2}\rho\left[1+\frac{\rho}{2\lambda}\right]\,, \tag{1}$$

where the brane tension λ relates the four dimensional Planck mass $M_{\rm p} \approx 10^{19}$GeV to the (fundamental) five dimensional Planck scale M_5 via [1]

$$M_{\rm p} = \sqrt{\frac{3}{4\pi}}\left(\frac{M_5^2}{\sqrt{\lambda}}\right)M_5\,. \tag{2}$$

This is actually a consequence of the cancellation of the negative bulk cosmological constant with the brane tension λ [15,16] that we have assumed. While the Friedmann equation is modified, the conservation of the energy momentum tensor remains valid on the brane. If we demand that successful nucleosynthesis takes place, then the second term proportional to ρ^2 has to play a negligible role at a scale $\sim \mathcal{O}(\text{MeV})$, corresponding to the era of Big Bang Nucleosynthesis (BBN). Therefore, we have to assume that the modified Friedmann equation paves a usual term on the right-hand side of Eq. (1), which is just linear in energy density [1]. This naturally leads to constraining the brane tension as $\lambda > (1~\text{MeV})^4$. A more stringent constraint on the brane tension can be obtained from the validity of the Newtonian gravity in $3+1$ dimensions on length scales smaller that 0.2 mm [20], which leads to constraining the brane tension as $\lambda > (1~\text{TeV})^4$ [21,22].

In our case the Universe exits from the non-conventional era when the energy density $\rho \sim \lambda$, this happens at a temperature $T_{transit} \sim \lambda^{1/4}$ in a radiation dominated Universe before BBN. At energy scales greater than the brane tension the thermal history of the Universe can be altered significantly, and some of the consequences have already been discussed in Ref. [3], where an upper bound $\lambda \leq (10^{10}\text{GeV})^4$ has been derived. This bound neglects the contribution from the KK spectrum, and thus the results of above reference holds good if the temperature of the Universe is below the normalcy temperature. Once thermalization of the final products of the inflaton has taken place, there could be an initial post-inflationary phase which is radiation dominated. The graviton production occurs due to thermal processes,

[1] We will frequently imply Eq. (1) to be a consequence of a non-conventional brane cosmology compared to the standard cosmology where $H = \sqrt{8\pi\rho/3M_{\rm p}^2}$.

such as photon-photon fusion, $\gamma + \gamma \to G_m$, where G_m corresponds to the KK graviton of mass m. This single process is generically Planck mass suppressed [22,23], such that the cross section goes as $\sigma_m \sim h^2/M_{\rm p}^2$, where h represents the dimensionless coupling constant of the KK mode. This can also be interpreted as the mode number density, which is given by the wave function of the graviton along the fifth dimension. The production of the KK modes occur at all temperatures because they are distributed continuously on mass. In the simple analogy of a hot radiating plate, one can imagine that a brane is a hot surface embedded in a cold space, it radiates gravitons with a spectrum which peaks around the brane temperature. The amplitude of the spectrum depends on the efficiency of the graviton production. In order to proceed with our calculation we need to know the density of states and for this purpose we have to find out the effective four dimensional wave function of these modes.

The actual calculation for the wave function of the KK graviton has been performed strictly in a static limit in Ref. [1,6]. The set-up is the following. Let us consider a 5 dimensional anti de-Sitter space (the bulk) where a flat brane of tension λ is located at $y = 0$; here y represents the infinite fifth dimension. The brane is devoid of any matter $\rho \sim 0$. The static metric which is a solution to the Einstein's equation of this set-up is given by

$$ds^2 = e^{-2\kappa|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,, \tag{3}$$

where the constant κ in the decaying warp factor relates the Planck and the fundamental scales by

$$\kappa \approx \frac{M_5^3}{M_{\rm p}^2}\,. \tag{4}$$

The above parameter also plays the role of the effective size of the extra dimension, since the KK contribution to gravitational interactions on the brane introduces a correction to the Newton's law which has a functional behavior of $\sim 1/\kappa r$ [1,21], which is similar to that of one large extra dimension [7,8] of size κ^{-1}. Notice, that the obtained Einstein's solution in Eq. (3), does not hold true if the metric has an arbitrary time dependence. The "static" KK graviton wave function in Gaussian-normal coordinates at the brane position is then given by [1,6]

$$|h_m(y=0)| = \frac{2}{\pi}\sqrt{\frac{\kappa}{m}}\frac{1}{\sqrt{J_1^2(\frac{m}{\kappa}) + N_1^2(\frac{m}{\kappa})}} \approx \begin{cases} \text{const.}\sqrt{\frac{m}{\kappa}} & \text{for} \quad m \ll \kappa \\ \text{const.} & \text{for} \quad m \gg \kappa \end{cases} \tag{5}$$

where m designates the mass of the KK mode (the fifth momentum of the graviton), and, const. $\sim \mathcal{O}(1)$. We notice, depending on the mass of the KK mode the projected wave function on the brane is different. The physical reason behind this effect is the presence of a volcano potential [1] felt by the gravitons which makes all the lighter modes weakly interacting with respect to the modes that are above the hight of the volcano potential, which is of the order of κ. In our case the situation is quite different, since we are certainly not in a static solution. We have a matter $\rho \neq 0$ on the brane. However, we presume that the structural form of the above equation remains intact except for some unknown constant factors. In what follows we shall assume the sanctity of Eq. (5) in our analysis.

At higher temperatures, $T > \kappa$, the total cross section for graviton production (summed over all possible final modes) is then given by the sum of both the regimes mentioned above in Eq. (5).

$$\sigma_{\gamma+\gamma\to G} \sim \frac{1}{M_{\rm p}^2}\int_0^T dn \equiv \frac{1}{M_{\rm p}^2}\int_0^\kappa \frac{m}{\kappa}\frac{dm}{\kappa} + \frac{\rm const.}{M_{\rm p}^2}\int_\kappa^T \frac{dm}{\kappa} \approx \frac{T}{\kappa M_{\rm p}^2}\,. \tag{6}$$

Notice, that in the above expression the main contribution to the cross section comes from the heavier modes rather than the lighter modes. The cross section then goes linearly in temperature, just as in the case of a single compact large extra dimension [8], where in general the same dependence goes as $(TR)^\delta$ for δ extra dimensions compactified on a torus. In fact in a good approximation the effective number of levels contributing at higher temperatures is given by $\sim (T/k)$. An interesting point to notice here is that at low temperatures, $T < \kappa$, the cross section goes as $\sigma \sim (T^2/\kappa^2 M_{\rm p}^2) \sim T^2/\lambda$, which actually mimics the result of two compact large extra dimensions. Here we stress that the KK modes are not distributed uniformly in energy scales. This is also the reason why the lighter states $T < \kappa$ induce a correction to the Newtonian potential as $1/(\kappa r)^2$, while the heavier states contribute to the correction as $1/\kappa r$ only [21,22]. From the five dimensional point of view this reflects that gravitons with a large fifth momentum are easier to produce since their energy is above the volcano barrier, whereas the lighter modes have to cross through such a barrier, thus their production is less efficient.

Now let us consider the evolution of the production of KK modes in a simple set-up by assuming that radiation is dominating the Universe. Our fist goal is to estimate the largest temperature that the photon bath can achieve without overproducing gravitons. That is what is known as the normalcy temperature. Below this temperature the cooling rate of the brane due to dissipating energy is less noticeable. As a first approximation to the problem we assume that the KK gravitons which have a momentum in the fifth direction have not gone very far away from the brane. This allows us to count their degrees of freedom as if they were lying on the vicinity of the brane. Therefore, the equation which governs the individual KK mode number density can be given by

$$\frac{dn_{G,m}}{dt} + 3Hn_{G,m} = \langle\sigma\ v\rangle_m\ n_\gamma^2, \tag{7}$$

Notice, that gravitons are actually escaping from the brane, such that our present approach actually overestimates their real number. However, the result will actually give us the safest temperature at which the expansion of the three spatial directions to the brane is not being affected by the KK gravitons which have being released into the bulk. In fact, as they propagate at most with the speed of light on the bulk, it certainly takes a while for them to go far from the brane and relive the brane from their influence. As the production is continuous they form a cloud around the brane that freely expands into the bulk as the brane cools gradually. Here we must remember that in Eq. (1), one has assumed that the main contribution to the Hubble expansion comes only from the brane matter. If a dense cloud of gravitons is surrounding the brane then their contribution to the expansion must also be taken into account. That may even restore for instance, the “dark radiation” term;

$\sim 1/a^4$, where a is the scale factor of the Universe. We remind the readers that this term has been neglected in Eq. (1). Such contributions shall remain negligible as far as the temperature of the thermal bath on the brane is lower than the normalcy temperature that we are about to calculate. By simply assuming the adiabatic expansion $a(t)T(t) = constant$, we can in fact simplify Eq. (7). While doing so we may also neglect the evolution of the individual mode and shall concentrate upon all possible KK states excited upto a given temperature. We obtain

$$\frac{d(n_G/n_\gamma)}{dT} = -\frac{\langle \sigma\ v \rangle n_\gamma}{H\ T}\,. \tag{8}$$

Once the KK states are excited they are no more in thermal equilibrium, we can integrate Eq. (8) while assuming that we are in a standard cosmological era such that $H^2 \propto \rho/M_{\rm p}$, we get

$$\frac{n_G(T)}{n_\gamma} = \mathcal{D}\ \frac{n_\gamma(T_{\rm r})\langle \sigma\ v\rangle}{H(T_{\rm r})}\,, \tag{9}$$

where $\mathcal{D}$ is the dilution factor which depends on the ratio of the number of relativistic degrees of freedom. In the Standard Model this ratio can be at most of order $\mathcal{D} \sim \mathcal{O}(10^{-2})$, if the maximum temperature is above ~ 1 GeV. The temperature $T_{\rm r}$ designates the largest temperature during radiation era which is also known as the reheat temperature of the Universe. We also take $v = 1$, henceforth. Now with the help of Eq. (6) and assuming that the relativistic particles dominate the Universe; $n_\gamma \sim T_{\rm r}^3$, we evaluate the right-hand side of Eq. (9). The ratio thus obtained can not exceed more than one at any later times in order to maintain the successes of the nucleosynthesis era and so we obtain a simple bound on $T_{\rm r}$, which is

$$T_{\rm r} \lesssim T_c \equiv \lambda^{1/4}\ = \sqrt{\kappa M_{\rm p}}\,, \tag{10}$$

where T_c represents the *normalcy temperature*. As mentioned it guarantees that below this temperature the production rate of gravitons is not efficient enough with respect to the number of photons, and thus, the Universe can be safely considered to be in the radiation dominated phase before BBN. Notice also that $T_c > \kappa$ is actually consistent with the assumptions in Eq. (6). The above temperature shall act as a test bed for any departure from the radiation era in a standard cosmology. Further, let us notice that this temperature is exactly the same as the transit temperature; $T_{transit} \sim \lambda^{1/4}$, that naively marks the transition from the non-conventional era to the standard Universe. This renders our analysis completely fool-proof.

Let us remark that in the interesting case where one assumes $\kappa^{-1} = 0.1$ mm, one obtains $T_c \approx 1$ TeV. This might render an extreme fine tuning in the inflaton coupling to the matter fields to reach a reheat temperature which is lower than the normalcy temperature. It is worth mentioning that unlike other relics, the KK modes are being radiated away from the brane to the infinite extra fifth dimension. Therefore, the KK modes actually escape all important cosmological constraints mainly coming from BBN. Notice, for instance, that unlike the case of those four dimensional fields with masses around TeV and Planck suppressed couplings, which decay very close to BBN era, in the present case a KK mode with the same mass will be far away from the brane by the time $\tau_{BBN} \sim 1$ sec. Therefore, such a mode should have lost all its interactions to the brane fields, and it will not decay back to the brane. Therefore, they are totally harmless.

In order to cross check our previous analysis, let us wonder what happens if the Universe prefers to thermalize during the non-conventional era while $H \propto (\rho/M_{\rm p}\sqrt{\lambda})$. We can repeat the same procedure. An important point to mention is that the structural form of Eq. (9) shall remain intact in our case [3]. Now, as H goes like T^4, the ratio in Eq. (9) becomes temperature independent. This tells us that the KK modes have already saturated the photons number density. This result is consistent with our assumption.

It is worth mentioning that the above analysis actually does not preclude the possibility of thermalizing the Universe during the non-conventional era, or, in general above the normalcy temperature given by Eq. (10). If the Universe thermalizes at temperatures larger than the normalcy temperature after the inflaton has decayed, then the initial radiation bath might be able to excite a larger number of KK gravitons overpassing the photon density. This might render the Universe in a phase where the energy stored in the relativistic species is quickly being released into the bulk in the form of KK gravitons, and eventually they decouple at some point from thermal history. The effective KK number density is then substantially reduced paving a radiation dominated phase, which must be restored at least before $\sim \mathcal{O}(1{\rm Mev})$. At higher temperatures the effective number of states one can excite follows as T/κ for temperatures $T \gg \kappa$, from Eq. (5) and Eq. (6).

Here we make an important remark on the difference between the cosmological models of a single brane with that of a compact large extra dimensions. In the compact large extra dimensions it has been noticed that the normalcy temperature of the Universe has to be $\leq$ $(1-100)$ MeV in order not to over produce the KK gravitons if the fundamental scale which is 4+2 dimensional gravitational constant is $\sim$ TeV [7,8]. In this case eventhough the produced gravitons have a momentum along the bulk, but the bulk is a compact space which allows these modes to hit the brane more frequently eventhough the size of the compact dimensions are as larger as millimeters. This means that these KK modes do not decouple from the brane, but their presence is felt physically on the brane. Unlike the infinite dimension case these modes in compact extra dimensions eventually decay on the brane matter which puts severe constraints on their number density. It has been shown that the reheat temperature of the Universe must be lower than the normalcy temperature. In order to achieve this one must promote the inflaton field as a bulk field, whose decay products reheats the Universe. It has also been noticed that in order to provide a dynamical mechanism to stabilize the extra compact dimensions and to inflate the $3+1$ spatial dimensions which could also provide the right amount of the observed density perturbations in the Universe, one needs to promote the inflaton field to the higher dimensions [24,25]. This inevitably leads to the Planck mass suppressed couplings between the inflaton and the Standard Model fields which resides only in our world. As an important consequence of this the reheat temperature of the Universe is always below the normalcy temperature and it is roughly given by $\sim (1-10)$ MeV. This is precisely the reason why such a low reheat temperature is able to prohibit any possibility of a KK domination just before nucleosynthesis, eventhough the temperature of the Universe at that time is larger than the mass gap of the discrete KK states which is given by an inversely proportional to the size of the extra dimension. However, in the case of a single brane the situation is quite different. First of all there is no compelling reason why the

Universe must thermalize into a radiation bath below the normalcy temperature T_c defined in Eq. (10), especially since the inflaton field may reside on the brane itself [26]. This leads to a natural question of what actually happens if the Universe prefers to thermalize above this temperature. This is the topic we shall study next.

It is quite evident, that if the KK modes are produced in such a way that their number density overshoots the other relativistic species, then the evolution of the Universe would not be that of the radiation era and the radiation domination could not be recovered until the last of the KK modes in excess has leaked away far from the brane. This shall be regarded as a *final reheating temperature* denoted by T_{final}. A naive approach tells us that this occurs when the wavefunctions of the gravitons which is still moving away from the brane has at least moved a distance equivalent to the inverse mass of the last possible mode in excess. This distance then corresponds to the KK mass of order κ, since all the modes above this mass contributes to the effective number of relativistic degrees of freedom. Thus, $m \sim \kappa$ defines our physical mass of the KK mode. Notice, it also means that the cloud formed due to these gravitons have moved away a distance equivalent to the effective size of the fifth dimension, such that the gravitational interactions to the brane fields become subleading. As the cloud can expand at most at the speed of light since gravitons are actually massless in five dimensions, the decoupling from the brane matter takes place at a time scale given by

$$\tau_{dec} \sim \kappa^{-1} \,. \tag{11}$$

When this happens the Universe comes back to a radiation dominated with a standard cosmology. Therefore, we estimate the final reheating temperature by equating $H \sim T^2/M_{\rm P}$ and κ. It is interesting to note that one obtains exactly the same value as the normalcy temperature,

$$T_{final} = \lambda^{1/4} \ . \tag{12}$$

This we recognise as the largest temperature a brane world can have in the radiation dominated era if the Universe thermalizes before the normalcy temperature.

An alternative explanation can be illustrated by calculating the cooling rate of the relativistic thermal bath. As the KK mode leaves the brane, the brane looses the energy at a rate given by [19];

$$\frac{\dot{\rho}_\gamma}{\rho_\gamma} = -\frac{\langle\sigma E\rangle n_\gamma^2}{\rho_\gamma} \approx -C\frac{\rho_\gamma}{\kappa M_{\rm P}^2} \ , \tag{13}$$

where the dimensionless constant factor $C \sim 0.1$ is given in terms of the distribution functions of the fields of the thermal bath [19]. One can easily integrate the above equation to obtain the time scale when the brane density has dropped down to $\rho_\gamma = \lambda$. One obtains the decoupling time scale upto an order one numerical factor,

$$\tau_{dec} \sim \frac{\kappa M_{\rm P}^2}{\lambda} = \kappa^{-1} \ . \tag{14}$$

This is a familiar result which we have obtained earlier.

We notice that the cloud of gravitons that surrounds the brane may modify the thermal history of the brane. Indeed, when the density of gravitons around the brane is not negligible, then the Hubble expansion becomes a function of the fifth dimension [4]

$$H^2(y) \approx \frac{(\rho_B - \Lambda)}{6M_5^3} + \left(\frac{a'}{a}\right)^2 , \tag{15}$$

Where Λ is the bulk cosmological constant, ρ_B is the bulk matter density and prime denotes the derivative with respect to y. At the position of the brane the last term in Eq. (15) along with Λ term reduces to a ρ squared contribution, where ρ is matter density on the brane as depicted in Eq (1). In order to understand the dynamics, one requires a profile of the graviton density within the cloud. Besides this, it has also been argued that the energy which has been radiated into the bulk might form a black hole. Such a scenario if happens introduces an extra contribution to the expansion which goes as $\sim 1/a^4$ [6,19]. As Ref. [19] argues, the presence or formation of a black hole in the bulk is important only when $T \sim M_5$. To avoid such a $\sim 1/a^4$ contribution the maximal temperature of the initial thermal bath on the brane should be smaller than M_5, which is the natural cut-off. However, we are much below this temperature and under this assumption we may justify the above expression Eq. (15).

In order to have a very rough estimation of the expansion due to the presence of the cluod, we assume that the energy density of the Universe is governed by the bulk density of relativistic modes. This is equivalent to the assumption of demanding $H^2 \approx \rho_B/M_5^3$ in Eq. (15). The temperature of such a bath is obviously larger than the normalcy temperature given by Eq. (10). To a good approximation we may consider the five dimensional graviton density to be uniform within distance κ^{-1}. This sets up a scale for the graviton cloud, which determines the bulk energy density as $\rho_B \approx \kappa\rho_{KK}$, where ρ_{KK} is an effective four dimensional KK density. This gives an uniform expansion rate parallel the brane, which reads

$$H^2 \approx \frac{\rho_{KK}}{M_{\rm P}^2} . \tag{16}$$

From the effective four dimensional point of view the situation mimics that in Ref. [9]. The temperature dependence of the effective density is determined by

$$\rho_{KK} \approx \left(\frac{T}{\kappa}\right) T^4 . \tag{17}$$

The term in a bracket corresponds to the relativistic degrees of freedom which is the number of KK modes that can be estimated from the wave function Eq. (5). Notice, from the five dimensional point of view this only tells us that $\rho_B \sim T^5$, as it should be expected on purely dimensional grounds. Given this, the Hubble parameter of the parallel directions to the brane is now modified to

$$H(T) \sim \frac{T^{5/2}}{\kappa^{1/2} M_{\rm p}} . \tag{18}$$

Let us point out that the rate of expansions is actually faster compared to the standard behavior $H \propto T^2/M_{\rm p}$. This can also be understood from purely five dimensional point of view by inspecting Eq. (15). We notice that the bulk energy density ρ_B is only suppressed by the fundamental scale rather than the Planck scale, which is an outcome of a single brane set-up.

Let us conclude with some remarks. The evolution of the early Universe in a single brane cosmology could be quite different than naive expectations. In order to solve homogeneity and the flatness problem one requires a phase of inflation in these models. Inflation might occur in the non-conventional era [26], or, in a conventional era. Depending on this initial phase the Universe might thermalize in a different way. The thermalization process inevitably renders the Universe as a radiation dominated era, either in a non-conventional era, or, in a standard era. In the former case the Universe undergoes inevitably through a KK graviton overproduction phase. During this period the energy of the photon bath is quickly transfered into the bulk in the form of gravitons which leave the brane once they are produced. This can be recognized as a cooling of a brane, which comes to an end only when the temperature of the radiation bath has dropped down the normalcy temperature. This happens well after the standard ρ behavior of H^2 has been established. The transition to the standard cosmology takes place when the last of the KK modes in excess has been dissipated from the brane a distance larger than the effective size of the fifth dimension κ^{-1}. This estimates the final reheat temperature $T_r \approx T_c \sim \lambda^{1/4}$. If thermalization occurs already in the standard cosmology, the initial radiation dominated phase shall be maintained. We conclude by saying that the temperature associated with this threshold can be safely thought as being the largest temperature of the Universe in a radiation era. It is worth noticing that since the KK modes escape into the bulk, they do not posse any serious problem for nucleosynthesis.

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